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## The effects of SHM system parameters on the value of damage detection information

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### Abstract

This paper addresses how the value of damage detection information depends on key parameters of the Structural Health Monitoring (SHM) system including number of sensors and sensor locations. The Damage Detection System (DDS) provides the information by comparing ambient vibration measurements of a (healthy) reference state with measurements of the current structural system. The performance of DDS method depends on the physical measurement properties such as the number of sensors, sensor positions, measuring length and sensor type, measurement noise, ambient excitation and sampling frequency, as well as on the data processing algorithm including the chosen type I error for the indication threshold. The quantification of the value of information (VoI) is an expected utility based Bayesian decision analysis method for quantifying the difference of the expected economic benefits with and without information. The (pre-)posterior probability is computed utilizing the Bayesian updating theorem for all possible indications. If changing any key parameters of DDS, the updated probability of system failure given damage detection information will be varied due to different indication of probability of damage, which will result in changes of value of damage detection information. The DDS system is applied in a statically determinate Pratt truss bridge girder. Through the analysis of the value of information with different SHM system characteristics, the settings of DDS can be optimized for minimum expected costs and risks before implementation.

### 1. Introduction

Over the last decade, Structural Health Monitoring (SHM) strategies and measurement techniques have been well developed and encompass various physical technologies (1) such as e.g. strain gauges, acceleration sensors, acoustic emission, ultrasonic technology, x-ray technology, in combination with a large variety of data analysis algorithms. However, there are often too many sensors and several may be incorrect (2). There is an urgent need for understanding the effectiveness of different sensor configurations, which is carried out in this paper through a value of information analysis (12) comprising a decision tree analysis, structural probabilistic models, consequences analysis as well as benefit and costs analysis associated with monitoring results through its service life.

This paper addresses how the physical measurement properties of the Damage Detection System (DDS), including the number of sensors and sensor positions, influence the value of DDS information. Through an example of a given truss bridge girder and a vibration-based damage detection algorithm (11), the number of sensors and their locations are selected based on the



dynamic loading and probabilistic model. Based on the sensor layout, the probability of damage indication is calculated. The value of DDS information is consecutively calculated with different configuration of the DDS. The results of value of information analysis can be used to provide a base for sensor placement as part of the design for new structures to identify an economical and reliable approach for a better evaluation of the structural condition through optimal sensor configuration.

## 2. Value of information analysis

The quantification of the value of information (VoI) for deteriorated structures is rooted in Bayesian updating and utility-based decision theory, which having a certain format to quantify the expected utility increase of unknown information. It is comprising a decision tree analysis, structural probabilistic models, consequences analysis as well as benefit and costs analysis associated with monitoring results through its service life. In general, the VoI can be found as the difference between the maximum expected utility obtained in (pre-)posterior analysis and the maximum expected utility obtained using only prior information, which means that a value to a piece of information can be assigned as the difference between expected utilities of the optimum decisions with and without that information (3).

$$Vol_i = \max_i E_{Z|i} \left[ \max_a E''_{\theta|Z} [u(\mathbf{i}, \mathbf{Z}, \mathbf{a}, \boldsymbol{\theta})] \right] - \max_a E'_\theta [u(\mathbf{a}, \boldsymbol{\theta})] \quad (1)$$

Where  $\mathbf{i}$  is the choice of information strategy,  $\mathbf{Z}$  is the possible outcomes,  $\mathbf{a}$  is the choice of the action and  $\boldsymbol{\theta}$  is the system states,  $u$  is the utility,  $E$  is the expected value.

## 3. Structural system characteristics

For a component  $i$ , failure occurs when the external load  $S_i$  exceeds the resistance  $R_i(t)$  due to an increase of damage and resistance degradation. Considering the resistance model uncertainties  $M_R$ , and the loading model uncertainty  $M_S$ , the probability of a general series-parallel system failure  $P(F_S)$  can be written as:

$$P(F_S) = P \left( \bigcap_{i=1}^{n_i} \bigcup (M_R R_i(t) - M_S S_i) \leq 0 \right) \quad (2)$$

The limit state function  $g_i(\mathbf{X}, \mathbf{D}) \leq 0$  represents the failure of component  $i$ . The vector of the system performance random variables  $\mathbf{X}$  then comprises the components resistance, loading and uncertainties. The vector of the system degradation random variables  $\mathbf{D}$  contains the collection of the deterioration states for all components. Monte Carlo simulation can be used to calculate the cumulative probability of system failure  $P(F_S)$  throughout the service life.

$$g_i(\mathbf{X}, \mathbf{D}) = M_R R_i(t) - M_S S_i(t) \quad (3)$$

For general corrosion and fatigue deterioration, the deterioration model (4, 5) can be written as:

$$D_i(t) = \alpha(t - T_j) \quad (4)$$

Where  $D_i(t)$  is the time-variant continuous damage state for component  $i$  at time  $t$ ,  $\alpha$  is annual deterioration rate,  $T_j$  is the deterioration initiating time at time  $j$  for element  $i$ . The resistance is degraded following:

$$R_i(t) = R_{i,0}(\Delta_i - D_i(t)) \quad (5)$$

Where  $R_i(t)$  is the time variant resistance for component  $i$ ,  $R_{i,0}$  is the initial resistance of element  $i$ ,  $\Delta_i$  is the damage limit of component  $i$ . The resistance is continuously reduced due to the accumulated damage evolution.

#### 4. SHM system and algorithm characteristics

Ambient vibration monitoring aiming for damage detection is one of the most known SHM techniques. Associated damage detection methods are based on the fact that damage can influence the structural stiffness, and thus the modal parameters (modal frequencies, damping ratios and mode shapes), which characterize the dynamics of the structure. Vibration measurements can indicate changes of dynamic characteristics of the structure and thus the states of damage. As an example, the statistical subspace-based damage detection method (11) is used in this study.

Based on ambient vibration measurements from a (healthy) reference state and measurements from the current system, this method computes a test statistic that compares both states. This test statistic is a random variable that is  $\chi^2$  distributed, having a central  $\chi^2$  distribution in the reference state and a non-central  $\chi^2$  distribution in the damaged state. The theoretical properties of the distributions are known and depend on the damage. A threshold is set up in the distribution of the reference state for a desired type I error for a decision between reference and damaged states: if the test statistic is below this threshold the structure is recognized as healthy, if it is above this threshold it is indicated as damaged. Based on the threshold and the theoretical properties of the  $\chi^2$  distribution for any given damage, the probability of indication of such a damage can be calculated, without using measurement data (6).

The performance of the DDS method depends, amongst others, on the following properties: 1) Measurement properties, which are related to the number of sensors, sensor positions, type of sensors, sampling frequency  $f_s$  and measurement length; 2) Stochastic system properties, such as properties of the ambient excitation (white noise) and measurement noise; 3) Type I error for indication threshold. In this paper, the influence of the number of sensors and sensor positions is examined.

#### 5. Bayesian updating

Based on the damage state, the DDS can provide the indication or no indication of damage. The respective probabilities of indication or no indication can be evaluated a priori with the chosen DDS method (6). The updated probability of system failure if given no indication of damage  $P(F_s|\mathbf{D}, \bar{I})$  can be calculated through Bayesian updating (7):

$$P(F_s|\mathbf{D}, \bar{I}) = \frac{P(\bar{I}|F_s, \mathbf{D})P(F_s, \mathbf{D})}{P(\bar{I})} = \frac{P(F_s|\mathbf{D} \cap \bar{I})}{P(\bar{I})} = \frac{P(g_s \leq 0 \cap g_u \leq 0)}{P(g_u \leq 0)} \quad (6)$$

Where  $P(\bar{I})$  is the probability of no indication,  $P(\bar{I}|F_s, \mathbf{D})$  is the probability of no indication given damage failure.  $P(F_s, \mathbf{D})$  is the probability of failure given damage, which can be calculated through Equation 2,3 and 4. To solve the Equation 6, the joint function of two limit states is computed. The limit state function of system failure  $g_s \leq 0$  can refer to Equation 3. The probability of no indication of detecting damage  $P(\bar{I})$  can be calculated by integrating in the region which is defined by the limit state function  $g_u \leq 0$ . The limit state function  $g_u$ , shown in Equation 7 is defined as the difference between the probability of indication given damage  $P(I|\mathbf{D})$  and a uniformly distributed random variable  $u$  (8).  $P(I|\mathbf{D})$  can be calculated through realization of damage state.

$$g_u = P(I|\mathbf{D}) - u \quad (7)$$

## 6. Example

### 6.1 Probabilistic model

The structural system performance of the Pratt truss bridge girder (shown in Figure 2) builds upon a series system and is coupled with time-variant damage models describing continuously the deterioration process and structural resistance degradation throughout the service life. The probability of system failure  $P(F_s)$  can be written as:

$$P(F_s) = P\left(\bigcup_{i=1}^{n_i} (M_R R_{i,0}(\Delta_i - \alpha(t - T_j)) - M_s S_i) \leq 0\right) \quad (8)$$

For the series system, the system limit state function is the minimum of the components limit state function:

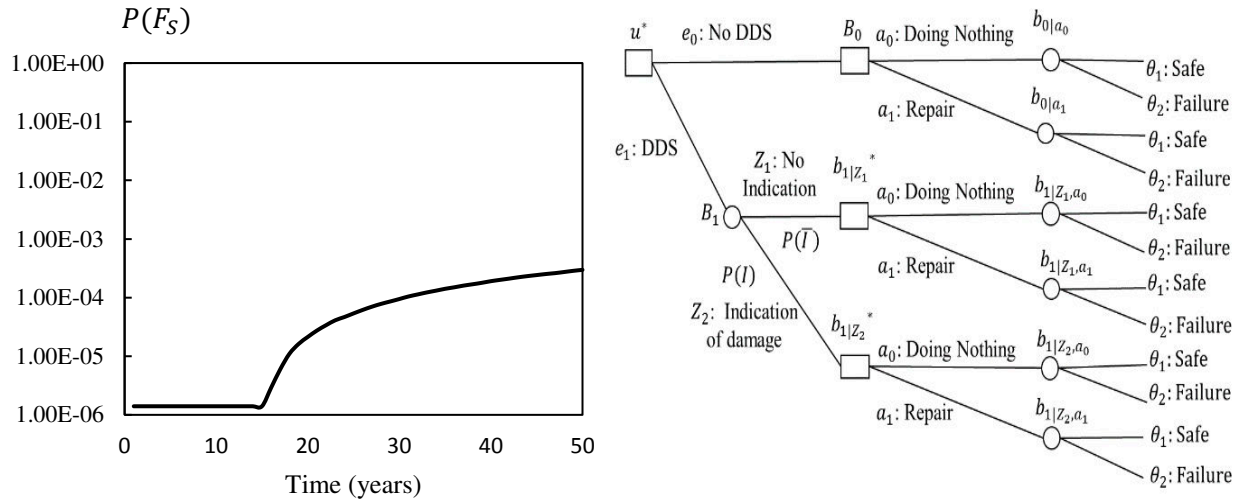
$$g_s = \min_{i=1 \text{ to } n_i} (M_R R_{i,0}(\Delta_i - \alpha(t - T_j)) - M_s S_i) \quad (9)$$

The mean of the initial resistance  $R_{i,0}$  is calibrated to a probability of system failure of  $10^{-6}$  disregarding any damage. The initial probability of failure is associated to large consequence of failure and small relative cost of safety measures (9). The input parameters of the probabilistic model are shown in Table 1. The expected prior probability of system failure during service life (50 years) is shown in Figure 1(left).

**Table 1 Input parameters of the probabilistic model**

Random variable	Distribution	Mean	Standard deviation
Loading $S_i$	WBL	3.50	0.1
Model uncertainty $M_s$	LN	1.00	0.1
Component resistances in undamaged state $R_{i,0}$	LN	Calibrated	0.1

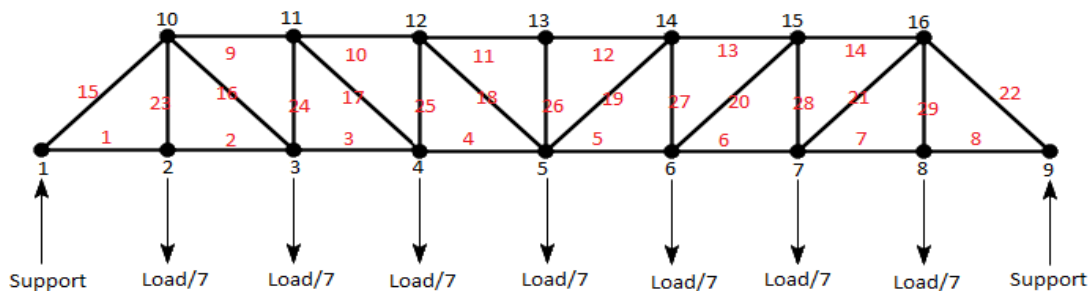
Model uncertainty $M_R$	LN	1.00	0.1
Damage limit $\Delta_i$	LN	1.00	0.3
Annual deterioration rate $\alpha$	Deterministic	0.00001	
Deterioration initiating time $T_j$	Deterministic	15	
Coefficient resistance correlation $\rho_R$	Deterministic	0.5	
Coefficient damage correlation $\rho_D$	Deterministic	0.5	



**Figure 1** Prior probability of system failure during service life (left); Illustration of decision tree combining a prior decision analysis and a pre-posterior decision analysis (right).

## 6.2 DDS sensor configuration

Two scenarios of sensor configurations are modeled, shown in Table 2. For the reference scenario, the damage detection system is modelled with the acceleration sensors located in node 12, 13, 14 of the truss in Y-direction recording the response for the DDS algorithm. Based on the dynamic structural system model, a reference dataset of length  $N = 10000$  at a sampling frequency of 50 Hz is simulated in the undamaged state. Ambient excitation (white noise) is assumed at all degrees of freedom whose covariance is the identity matrix. Gaussian measurement noise is added on the resulting accelerations with 5% standard deviation at each sensor signal. The type I error for indication threshold is set as 1%. In scenario (a) the number of sensors varies from 1, 3, 5 to 8. In scenario (b) the sensor positions are varied with a fixed number of three sensors.

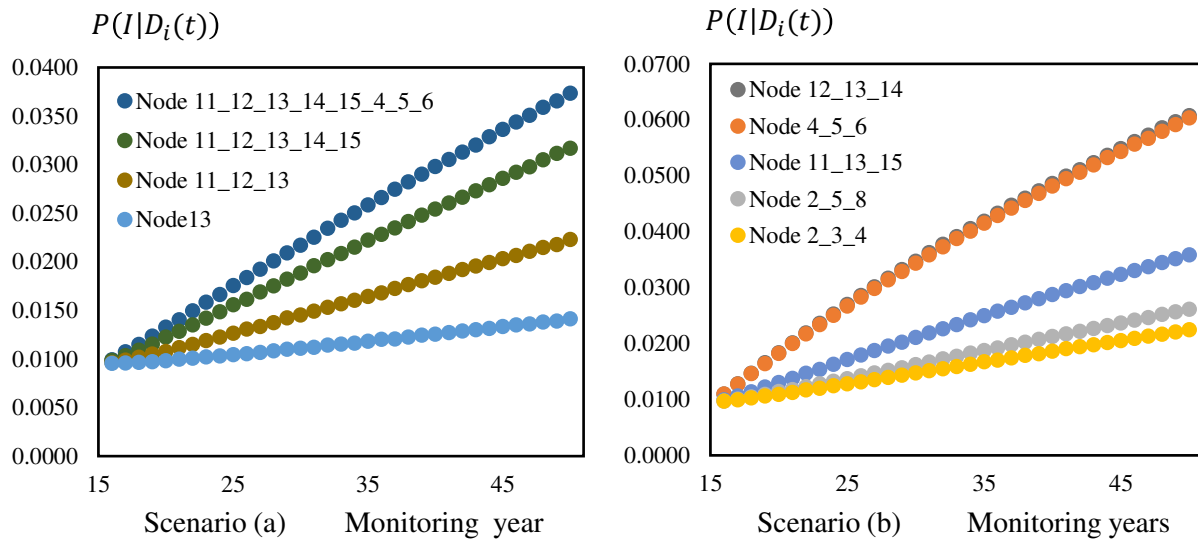


**Figure 2** Pratt truss bridge girder

**Table 2 Sensor configuration**

		Number of sensors	Sensor Node location	Measurement noise	Type 1 error
Base scenario	Case 0	3	12,13,14	5%	1%
Scenario (a)	Case 1	1	13		
	Case 2	3	11,12,13		
	Case 3	5	11,12,13,14,15		
	Case 4	8	11,12,13,14,15,4,5,6		
Scenario (b)	Case 5	3	4,5,6		
	Case 6		2,5,8		
	Case 7		2,3,4		
	Case 8		11,13,15		

The DDS is implemented in a particular year and the monitoring lasts for one year. The probability of damage indication in each operation year is shown in Figure 3.

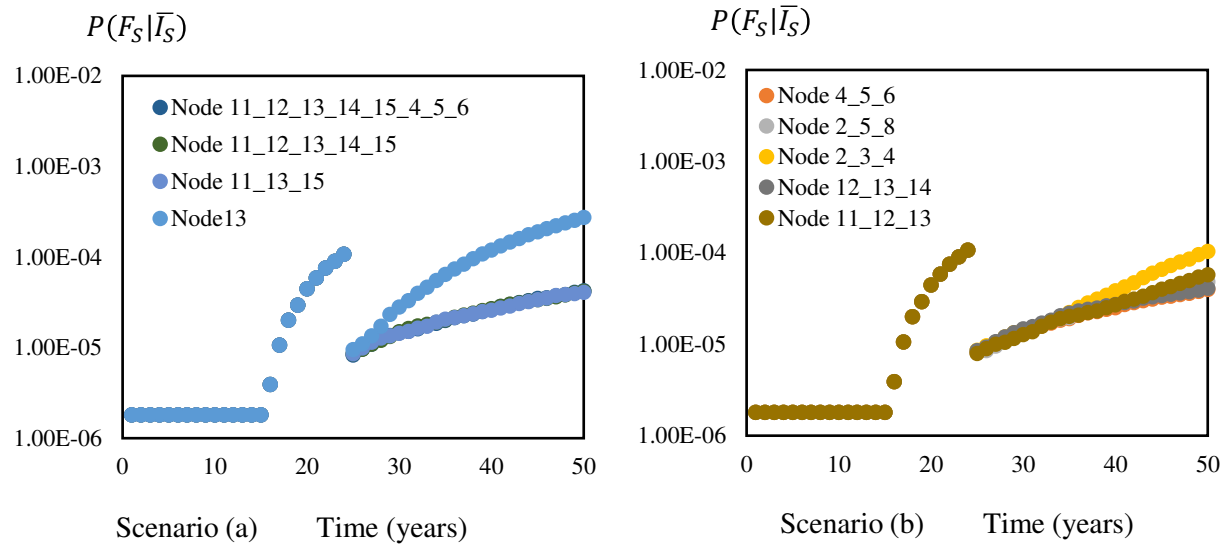


**Figure 3 PoI with changes of number of sensors during service life (a); PoI with changes of sensor location during service life(b)**

From Figure 3 (a) shows that the probability of damage indication (PoI) will increase with the number of sensors from 1,3,5 to 8 accordingly, which indicates that it is more probable to detect the damage with more sensors. When installing three sensors (Figure 3 (b)) it is observed that the closer the sensor location is to the weakest components 11 and 13, the larger the PoI will be. The maximum PoI during the service life will be the case when the sensors are located in nodes 12, 13 and 14. It is noted that due to the symmetry of the truss bridge girder, the sensor positions in node 4,5 and 6 will lead to the same PoI curve as for the sensor locations in 12, 13 and 14.

### 6.3 Bayesian updating

The updated probability of system failure given damage detection information is computed following section 5. The results are shown in Figure 4. When increasing the number of sensors, the updated probability of failure will be much lower than the case with only one sensor. However, it can be seen that the (pre-)posterior probability of system failure will not be lower if installing more sensors. Instead, the curve of the (pre-)posterior probability will almost be similar if more than one sensor is installed, which can be explained that only sensor in a specific position provides full function. When installing three sensors, if the sensor positions are far away from the weakest component 11 and 12, such as in node 2,3,4, the updated probability of failure will be larger in towards the end of the service life.



**Figure 4 (Pre-)Posterior probability of failure when implementing DDS at year 24 with different sensor locations during service life(a); (Pre-)Posterior probability of failure when implementing DDS at year 24 with different number of sensors during service life (b)**

## 6.4 Value of information analysis

The illustration of the decision tree is shown in Figure 1 (right). The costs model is shown in Table 3. The cost of repair  $C_R$  is increased with time due to the fact of increased severity of damage, which follows (10).  $T_{SL}$  is the service life,  $t$  is the repair year. When the bridge is repaired, it performs as a new one with the same probabilistic characteristics as originally, in which probability of system failure after repair equals to  $10^{-6}$ . The system is required to take repair action when the probability of failure exceeds of  $10^{-4}$  (9).

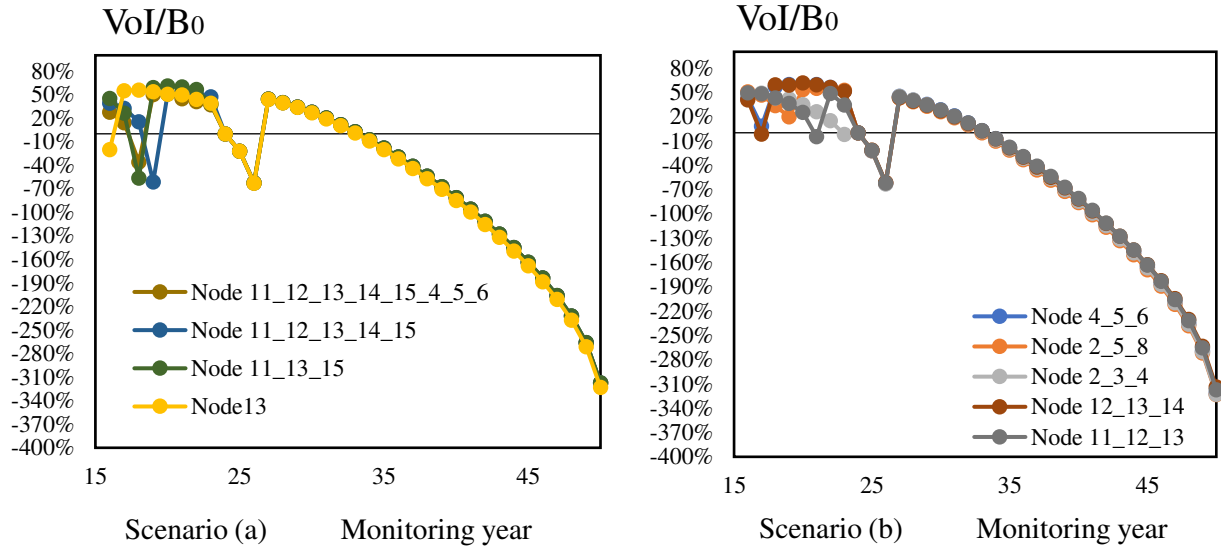
$$C_R = \frac{C_I}{T_{SL} + 2 - t} \quad (10)$$

**Table 3. Costs model**

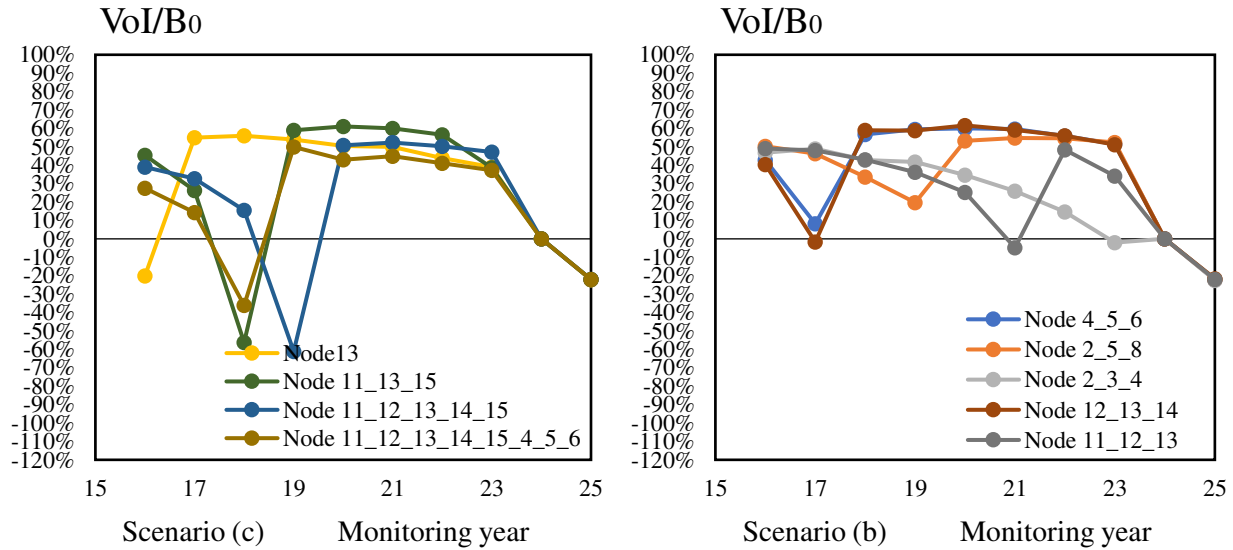
Discount rate $r$	Investment cost $C_I$	Failure cost $C_F$	Localization cost $C_{loc}$	DDS cost $C_{DDS}$
0.02	10	1000	0.1	0.1(per sensor)



The VoI analysis results depending on the DDS operation year are shown in Figure 5 and Figure 6, VoI/B<sub>0</sub> is the relative value of information, which is divided by the initial benefits and costs without that information.



**Figure 5 VoI/B<sub>0</sub> with different sensor locations during service life (a); VoI/B<sub>0</sub> with different number of sensors during service life (b)**

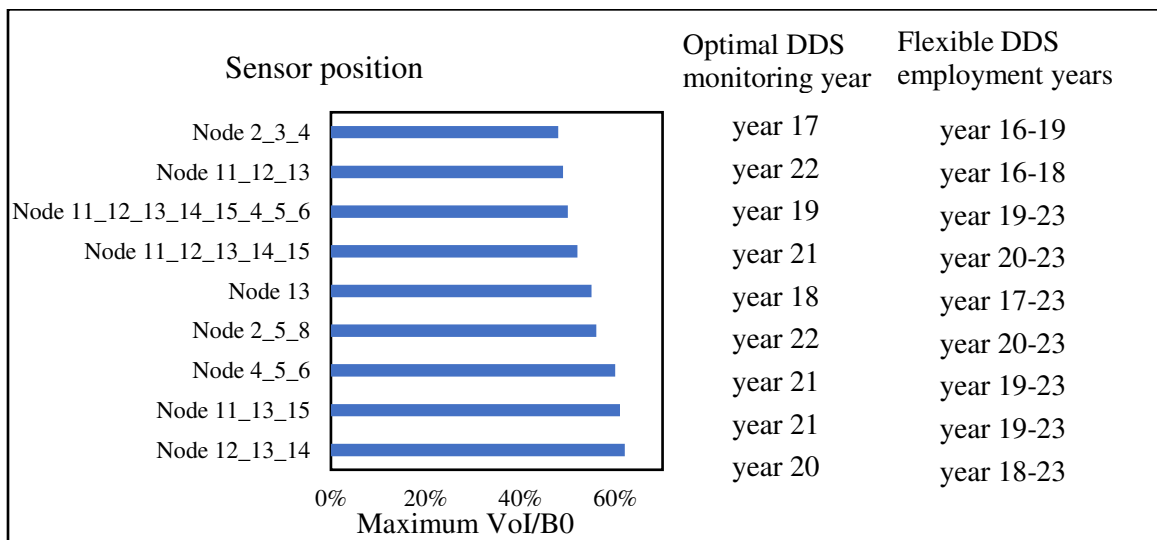


**Figure 6 VoI/B<sub>0</sub> with different sensor locations before year 25 (a); VoI/B<sub>0</sub> with different number of sensors during before year 25 (b)**

From Figure 5 and Figure 6, the VoI will generally first drop for early monitoring years due to the high total repair costs. Then it will increase due to the reduction of repair times. After reaching the peak, the VoI will decrease again due to the increase of failure risk and repair costs per time. From Figure 1 (left), it is shown that the probability of system failure will reach  $10^{-4}$  at year 24, so that

the VoI will be zero at that time because the system will always take repair action no matter implementing DDS or not. However, the VoI will stop dropping and increase again at year 26 due to only repair one time after that for whole service life. It will remain positive until year 33 and continuously drop until the end of service life due to the increase of failure risk and repair costs per time.

More sensors (Figure 6 (a)) will lead to more DDS costs, which will result in a lower VoI. The optimal period when all cases have a positive and minimal changes of VoI will be from year 20 to year 23. From Figure 6 (b), when the number of sensors are fixed, the closer the sensor location is to the weakest components 11 and 13, the larger the PoI will be, which results in a higher VoI. The maximum VoI during the service life will be the case when the sensors are located in nodes 12, 13 and 14 with 62% of relative VoI/B0 if applying DDS at year 20. This means that the total expected risk and costs can be reduced with this optimal sensor configuration and the employment of the SHM system in year 20. It is noted that due to the symmetry of the truss bridge girder, the sensor positions in node 4, 5 and 6 will lead to the same VoI curve as for the sensor locations in 12, 13 and 14. The summary of all the sensor configuration with maximum relative VoI/B0 and flexible DDS employment years is shown in Figure 7.



**Figure 7 Summary of optimal configuration and DDS employment time**

## 7. Conclusion

This study has shown that a one sensor system provides high value of information (between 39% and 56% of relative VoI/B0) for the longest time range (7 years from year 17 to year 23). This will allow for flexible DDS employment years in combination with minimum SHM system investment costs. System with a high number of sensors will have a positive value of information but in significantly less employment years. The sensor locations should be chosen with thorough consideration of the damage and failure scenarios of the structural system as only specific sensor locations near the highest utilized components lead to a high value of information. It should be

noted that we analysed a finite set out of many possible sensor configurations and there might be other configurations which may lead a slightly higher value of information.

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